# Anomalous delay in wave propagation and tunneling: A transition-elements analysis of the traversal time

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An alternative model for near-field propagation and optical tunneling is proposed following the lines of the path-integral method developed by Feynman, and in particular by using a transition-elements analysis. Such a model was able to account for the frequency dependency of delay-time results of an experiment involving microwave propagation in the near field using two horn antennas [A. Ranfagni *et al.*, Phys. Rev. E **66**, 036111 (2002)]. Furthermore, this approach is also capable of interpreting delay-time results as a function of the barrier width in a frustrated total internal reflection experiment performed at the microwave scale and in the optical region.

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In a previous paper [1], we reported on anomalous delay-time results in a microwave propagation experiment, which demonstrated a superluminal behavior strongly dependent on the frequency. These results were interpreted in the framework of a stochastic model and, in particular, according to a transition-elements analysis whose results were anticipated as Eq. (6) in Ref. [1]. The purpose of this work is to elucidate the attainment of such result and to demonstrate its capability of interpreting other traversal-time data as the ones obtained in tunneling experiments.

First, let us briefly recall the concept of transition elements. Feynman and Hibbs [2] introduced the transition elements with a general type of notation, and developed interesting relations between them: some of these relations, they stated, might well serve to generalize some laws of quantum mechanics. As noticed in Ref. [2], "it is difficult to understand transition elements on the level of intuitive physics." The simplest way is to consider them as weighted averages, where the weighting function $\exp(iS/\hbar)$, $S$ being the action, is a complex quantity, so the result too is complex, and it is not an "average" in the ordinary sense. In spite of this, the adoption of transition elements revealed its usefulness dealing with several problems: some cases of applications of this kind to the problems that we are considering have been reported in the literature [3,4].

In this work, we shall simply focus our attention on a result valid for quadratic action integrals that can be viewed as a starting point for an alternative, original model for tunneling and classically allowed processes, within the limit of high frequencies. This approach enables us to take into account dissipative effects, which are always present at the macroscopic level, and to provide a tool for interpreting the frequency dependency of delay-time results of previously performed experiments that involve microwave propagation in the near field with two horn antennas [1,5]. The problem we are addressing is not that of macroscopic quantum tunneling, but rather that of wave equations with dissipation [6], whose connection with quantum-mechanical equations has been well established [7]. We have to note, however, that when we say "dissipation" in tunneling—but also in near-field situations—we are not dealing with a true dissipation, but rather with an imaginary quantity introduced in order to obtain the analytical continuation of the wave equation (see below).

In the following, we derive an expression for the real part of the transition element of the time, which can be interpreted as the traversal-time duration. Indeed, we recover an expression, which is very similar to the one already obtained by a stochastic approach to the problem [8]. An approach, which supplies for the delay time a complex quantity, the real part of which is directly related to the measurements, while the imaginary part is an "apparent time," which can be related to the attenuation of the waves, corresponding to semiclassical evaluations [9]. On this basis, a plausible interpretation was given of delay-time results in near-field propagation as a function of frequency [1] and barrier width in microwave and optical frustrated total internal reflection experiments considered here.

Let us assume that the action integral $S'$ describing the development of a system in the presence of dissipative effects can be written in the form

$$S' = S + \int f(t)x(t)dt, \tag{1}$$

with $S$ being an undisturbed quadratic action integral, $f(t)$ any arbitrary function of the time, and $x(t)$ a path. In accordance with a phenomenological approach to the dissipative effects [10], we can identify $f(t) = \eta\dot{x}(t)$, $\eta = 2ma$ being the dissipative constant, $m$ "the mass of the particle," and $a$ the dissipative parameter entering the telegrapher's equation [6].

The transition element of $S' - S$ is defined [2] as

 

$$\left\langle \exp\left[ (i/\hbar) \int f(t)x(t)dt \right] \right\rangle$$

$$= \int_a^b \exp\left\{ (i/\hbar)\left[ S + \int f(t)x(t)dt \right] \right\} \mathcal{D}x(t) \quad (2)$$

$$= \{\exp[(i/\hbar)(S'_{cl} - S_{cl})]\} \langle 1 \rangle_S, \quad (3)$$

where the symbol $\mathcal{D}x(t)$ means that we are dealing with a functional integration. Equation (3) is obtained as a consequence of $S$ being a quadratic form, $\langle 1 \rangle_S$ means the propagator from $x_a$ to $x_b$, that is, $\int_{x_a}^{x_b} \exp[(i/\hbar)S]\mathcal{D}x(t)$, assuming the initial and the final wave functions as Dirac $\delta(x)$ functions. Our goal is to evaluate the transition element of the trajectory $x(t)$, from which we can obtain the traversal time by dividing by the velocity. By differentiating Eq. (3) with respect to $f(t)$, we obtain [Eq. (7.69) in Ref. [2]]

$$\langle x(t) e^{(i/\hbar)\int f(t)x(t)dt} \rangle = \frac{\delta S'_{cl}}{\delta f(t)} e^{(i/\hbar)(S'_{cl} - S_{cl})} \langle 1 \rangle_S, \quad (4)$$

where the symbol $\delta$ means that we are considering a functional derivative. In the following development of the work, the transition element $\langle x(t) e^{(i/\hbar)\int f(t)x(t)dt} \rangle$ will be denoted with $\langle \tilde{x} \rangle$ for simplicity of notation.

It is well known that tunneling processes may be well described by means of a classical equation of motion, which visualizes the motion as occurring in an inverted potential and in imaginary time. The equation of motion is then deduced from that of a damped harmonic oscillator, to which we can assimilate our case taking the telegrapher's equation as basis of our analysis and $S$ to be quadratic [6]. However, in view of the forbidden character of the process, the analytical continuation into a complex plane has been considered. This continuation can also be obtained by replacing the damping parameter $a$ with $ia$ [7]. The equation of motion becomes ($t$ is now real)

$$\ddot{x}(t) + 2ia\dot{x}(t) + \omega_0^2 x(t) = 0, \quad (5)$$

which is easily integrated with the boundary conditions $x(0)=0$ and $\dot{x}(0)=v$, thus obtaining

$$\overline{x(t)} = e^{-iat} \frac{v}{\tilde{\omega}} \sin(\tilde{\omega} t) \quad (6)$$

with $\tilde{\omega} = \sqrt{\omega_0^2 + a^2}$. Equation (6) represents the classical path.

Variation in the action may be computed by working out the integration overtime from the initial instant $t=0$ to the final instant $t$:

$$S'_{cl} - S_{cl} = \eta \int_0^t \overline{x(s)}\, \overleftarrow{x(s)}\, ds. \quad (7)$$

By taking into account Eq. (6), we have

$$S'_{cl} - S_{cl} = \frac{miav^2}{\tilde{\omega}^2} \sin^2(\tilde{\omega} t) e^{-2iat}. \quad (8)$$

Let us now evaluate the first-functional derivative of the action $S'_{cl}$ with respect to $f(t)$ by applying Eq. (7.25) of Ref. [2]:

$$\frac{\delta S'_{cl}}{\delta f(t)} = -\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{f}} + \frac{\partial \mathcal{L}}{\partial f} \quad (9)$$

$$= \frac{1}{\eta}\frac{\partial \mathcal{L}}{\partial \dot{x}} = \bar{x} + \frac{1}{\dfrac{\partial \dot{x}}{\partial x}} \overleftarrow{x}. \quad (10)$$

The first term in Eq. (9) is zero with $\mathcal{L} = \eta x \dot{x}$ being the Lagrangian that accounts for the dissipative effects. By differentiating Eq. (6), we have $\overleftarrow{x} = e^{-iat} v[\cos(\tilde{\omega} t) - (ia/\tilde{\omega})\sin(\tilde{\omega} t)]$ and we obtain

$$\frac{\partial \dot{x}}{\partial x} = -\tilde{\omega}\tan(\tilde{\omega} t) - ia. \quad (11)$$

This analysis holds for small displacements from the classical path, that is, in the limit of $f \to 0$ and, in any case, we have to satisfy the stationary condition $d(S'_{cl} - S_{cl}) = 0$. By differentiating Eq. (8) and equating to zero,

$$d(S'_{cl} - S_{cl}) = \frac{miav^2}{\tilde{\omega}^2}[2\tilde{\omega}\sin(\tilde{\omega} t)\cos(\tilde{\omega} t) - 2ia\sin^2(\tilde{\omega} t)]e^{-2iat} = 0, \quad (12)$$

we obtain $\tan(\tilde{\omega} t) = -i\tilde{\omega}/a$, which, by substituting into Eq. (11), gives

$$\frac{\delta S'_{cl}}{\delta f(t)} = \bar{x} - \frac{ia}{\tilde{\omega}^2 - a^2} \overleftarrow{x}. \quad (13)$$

We now have the means to deepen our understanding of the ways, in which the transition element of $x$ changes with time.

By making use of Eq. (8), within the limit of high values of $\tilde{\omega}$, the exponential function $e^{i(S'_{cl} - S_{cl})/\hbar}$ in Eq. (4) can be developed in a power series limiting to the first-order term. Then, by using the identification [7], $mc^2/\hbar \leftrightarrow \tilde{\omega}$, we obtain

$$\langle \tilde{x} \rangle \simeq \left( \frac{\delta S'_{cl}}{\delta f(t)} \right)_{f \to 0} \left[ 1 - \frac{a}{\tilde{\omega}} \frac{v^2}{c^2} \sin^2(\tilde{\omega} t) e^{-2iat} \right] \langle 1 \rangle_S, \quad (14)$$

where, we recall, $\langle 1 \rangle_S$ stands for the propagator. By making use now of Eq. (13), and by taking the average value of the function $\overline{\sin^2(\tilde{\omega} t)} = 0.5$, for the real part of the transition element of $\tilde{x}$, we obtain

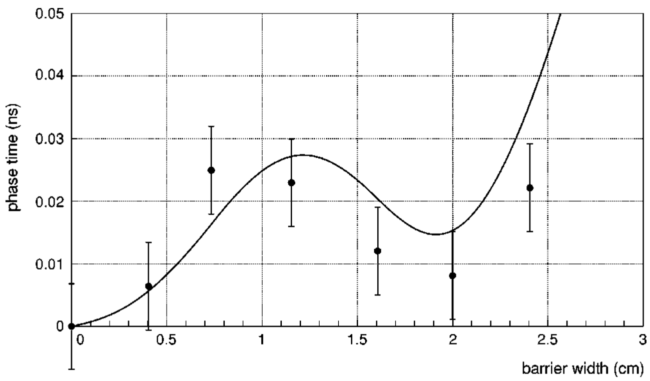


FIG. 1. Traversal- (or phase) time results as a function of the gap width $L$ between the two paraffin prisms in the case of the frustrated total internal reflection experiment at the microwave scale with a beam at 9.33 GHz after Ref. [12]. The continuous curve is deduced from Eq. (16), with a dissipative parameter $a=59\ \mathrm{ns}^{-1}$, and a propagator value of 0.5.

$$\mathrm{Re}\langle\tilde{x}\rangle \simeq \left[\bar{x}-\bar{x}\frac{a}{2\tilde{\omega}}\frac{v^2}{c^2}\cos(2at)-\bar{x}\frac{a^2}{2\tilde{\omega}(\tilde{\omega}^2-a^2)}\frac{v^2}{c^2}\sin(2at)\right]\times\langle 1\rangle_S . \qquad (15)$$

Under the same assumption as before, that is of high values of $\tilde{\omega}$, we can disregard the last term $\propto a^2/\omega^3$ in the square brackets of Eq. (15) and pass to time by dividing by the velocity $v$. We thus obtain an expression for the real part of the transition element of the time—which can be interpreted as real traversal time—namely [Eq. (6) in Ref. [1]],

$$\mathrm{Re}\langle t\rangle \simeq \frac{L}{v}\left[1-\frac{a}{2\tilde{\omega}}\left(\frac{v}{c}\right)^2\cos\left(2a\frac{L}{v}\right)\right]\langle 1\rangle_S , \qquad (16)$$

where we have identified $t$ with $L/v$, $L$ being the barrier width.

The propagator $\langle 1\rangle_S$ can be related to the attenuation of the waves, that is, $\langle 1\rangle_S \propto \exp(-2\pi L/\lambda)$, $\lambda$ being the wavelength [5]. It is difficult to evaluate the propagator, and an exact calculation can be performed only in a very limited number of cases. Nevertheless, the effect introduced is rather moderate. The resemblance of the result expressed by Eq. (16) to the one obtained in the stochastic approach [8] is now evident, since in this latter case, we obtained, in its simplified version, the expression $\mathrm{Re}\langle t\rangle \propto [1-\cos(2aL/v)]$.

Now, we intend to demonstrate that the model developed above is suitable for interpreting some of the available experimental results. First, we have considered the frequency dependence of the delay-time results of an experiment involving microwave propagation in the near-field region using two horn antennas. For the special type of waves involved (complex waves), this can be considered a case of pseudotunneling [5]. In Ref. [1], we reported a complete account of delay measurements as a function of the frequency performed by means of an experimental setup analogous to the one of Ref. [5]. The results showed an evident undulating shape with a periodicity that is almost independent of the frequency and oscillates between luminal and superluminal behavior [11]. The curves reported in Fig. 2 of Ref. [1] were obtained by calculating the expression $1-\cos(2\pi\nu L/v)+A$, derived from Eq. (16), by assuming $L/v=1.8$ ns, $a\simeq\omega/2 = 14$–$22\ (\mathrm{ns})^{-1}$, $\langle 1\rangle_S\simeq 1$, and $A\simeq 1$–$1.5$ ns, which accounts for the off zero due to the travel in the two horns. In this way, we obtained a reasonable description of the data.

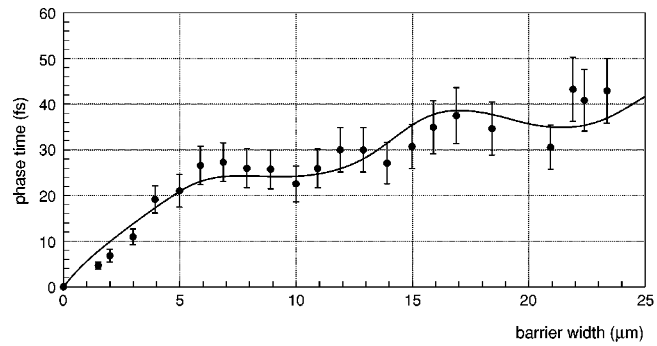


FIG. 2. Traversal- (or phase) time results as a function of the gap width $L$ between the two fused silica prisms in the case of the frustrated total internal reflection experiment at the optical scale with a beam at 3.39 $\mu$m after Ref. [9]. The continuous curve is deduced from Eq. (16), with a dissipative parameter $a = 0.050\ \mathrm{fs}^{-1}$, and a propagator value of 1.

The relation derived above, namely, Eq. (16), is also able to interpret delay-time results as a function of the barrier width in tunneling systems. Here as follows, we refer to experiments of frustrated total internal reflection that have been performed in both the microwave range (wavelength, $\lambda \simeq 3$ cm), and the visible region ($\lambda = 3.39\ \mu$m), where superluminal behavior was demonstrated. The results were obtained by measuring the shift of a beam traversing a barrier, in the first case, of a few centimeters between two paraffin prisms, while total reflection takes place in the first prism and evanescent waves originate in the gap [12]. In the second case, fused silica prisms were employed, and the gap was varied up to 25 $\mu$m [9]. Previously, we interpreted the delay-time results by using of a stochastic approach [12]. The model utilized fitted the experimental data well, allowing us to estimate also the dissipative parameter $a$, between 30–35 $\mathrm{ns}^{-1}$ in the microwave case. In Ref. [13], a slightly different stochastic approach, one that is derived from a Brownian-motion scheme, is described. Within that framework, delay-time measurements relative to frustrated total internal reflection experiments, in both the microwave and visible regions were interpreted. A plausible description of the trajectories inside the gap was also given.

Noting the similarity of the relation previously derived for the real part of the transition element of time to the one derived in the simplified stochastic model approach [8], we fitted delay-time results, as a function of the gap width, through Eq. (16) using $a$ and the propagator value $\langle 1\rangle_S$ as adjustable parameters. The velocity entering Eq. (16) was given by $v=c/\sqrt{n^2\sin^2 i-1}$, in the microwave case, $c$ being the light velocity in vacuum, and $i$ the incidence angle [12]. Figure 1 shows the experimental traversal-time results (circles with error bars) as a function of the gap width between the two paraffin prisms ($n=1.49$), with $i=60°$ as incidence angle. The critical angle is $i_c=42°$, and $\lambda \simeq 3$ cm. The curve (solid line) obtained by the theoretical model for $a=59\ \mathrm{ns}^{-1}$ and a propagator value $\langle 1 \rangle_S=0.5$, which corresponded to a moderate attenuation, provided a reasonable description of the data.

At the optical scale, we adopted an evaluation for the velocity that was derived from Eq. (2) of Ref. [9], thus allowing it to vary within the 0.6c–2.3c range [14]. Figure 2 shows the results relative to the optical experiment of Ref. [9], with $\lambda=3.39\ \mu$m, $n=1.409$, and the incidence angle $i=45.5°$, close to the critical angle $i_c=45.21°$. The curve obtained by the theoretical model for $a=0.050\ \mathrm{fs}^{-1}$, $\langle 1 \rangle_S=1$, gave a rather good description of the experimental data.

It seems, therefore, that the transition-element analysis adopted makes it possible to achieve an interpretation of the experimental results of delay time, in both allowed (free propagation in the near-field limit [1]) and forbidden (tunneling) processes, alternative to and even better than the former ones, even if in one sense, the present approach is leading again to the same kind of processes, namely, the stochastic ones. This appears to be a promising result, which deserves further investigations in an attempt to attain a deeper understanding of these complex phenomena.

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